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THEORETICAL FUNDAMENTALS OF MULTI-BODY METHOD

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Abstract

The general heat balance equation for multi-body system represents a stable system, with different time constants. The mathematical basis of the multi-body method is also given.

Keywords: multi-body method

Introduction

The basis of the multi-body theory were formulated previously [1–5]. The multi-body method based on this theory is very useful for analysis of various heat effects in calorimeter [3, 4, 6, 7]. It was also applied for determination of thermokinetics [6–9], e.g. heat power as the function of time for describing any heat effects.

The big advantage of this method, in comparison with existing methods based on electrical-heat analogy [10], is the application of description of heat phenomenon together with the heat exchange theory and control theory. Many scientific works show its advantages, giving unique stables solution.

The aim of this work is to prove, that the general heat balance equation used in multi-body method represents a stable system, the dynamics of which is described by real, positive, various time constants of the system.

General assumptions

In the multi-body method, the calorimetric system is treated as a system of bodies (domains) placed in environment of constant temperature. The bodies (domains) are treated as capacity type elements and each of them is characterized by constant heat capacity and uniform temperature in the total body volume. Between the defined bodies the heat exchange can occur by dividing media, assuming that heat capacities of these media are negligible.

The amount of heat exchanged between any two bodies or between the body and environment by dividing media is proportional to their temperature differ-

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ences and proportionality coefficient corresponds to the heat loss coefficient. In each defined body the heat source and temperature sensor may occur.

According to these assumptions, the behaviour of real calorimetric system is described by the following set of linear differential equations with constant coefficients [1]

$$C_j \mathrm{d}T_j(t) + G_{0j}[T_j(t) - T_0(t)]\mathrm{d}t + \sum_{i=1}^{N} G_{ij}[T_j(t) - T_i(t)]\mathrm{d}t = \mathrm{d}Q_j(t)$$

$$j = 1, 2, \ldots, N; \quad i \neq j \tag{1}$$

where: N – number of bodies; C_j – heat capacities of bodies; G_{0j} – heat loss coefficients between the bodies and environment; G_{ij} – heat loss coefficients between the bodies; $T_j(t)$ – functions describing the changes of temperatures in time of the defined bodies; $T_0(t)$ – environment temperature; $\mathrm{d}Q_j(t)$ – amount of heat delivered in time $\mathrm{d}t$ in the given body. The Eq. (1) are called general heat balance equation.

Uniqness of solution of heat balance equations

The heat balance equations for multi-body system give a set of linear differential equations, which corresponds in matrix form to the following equations:

$$C\frac{\mathrm{d}T(t)}{\mathrm{d}t} = B\cdot T(t) + r(t) \tag{2}$$

where C is diagonal matrix, which elements are heat capacities of the detailed bodies; elements b_{ij} of matrix B are

$$b_{ij} = G_{ij}; \quad i, j = 1, 2, \ldots, N; \quad i \neq j \tag{3}$$

$$b_{i0} = G_{i0}; \quad i = 1, 2, \ldots, N \tag{4}$$

$$b_{ii} = -\left(G_{i0} + \sum_{j=1}^{N} G_{ij}\right) = -\sum_{j=0}^{N} G_{ij}; \quad i=1, 2, \ldots, N; \quad i \neq j \tag{5}$$

and

$$r(t) = P(t) + W(t) \tag{6}$$

where the vectors $P(t)$ and $W(t)$ are defined as follows

$$W^{\mathrm{T}}(t) = [W_1(t), W_2(t), \ldots, W_N(t)] \tag{7}$$

$$P^{T}(t) = T_0(t)[G_{10}, G_{20}, \ldots, G_{N0}] \tag{8}$$

Multiplying leftside both sides of Eq. (2) by C^{-1}, we obtain

$$\frac{dT(t)}{dt} = A \cdot T(t) + D \cdot r(t) \tag{9}$$

where

$$A = C^{-1} \cdot B; \; D = C^{-1} \tag{10}$$

Using relationship

$$C^{-1} = C^{-1/2} \cdot C^{-1/2}; \; C^{-1/2} \cdot C^{1/2} = I \tag{11}$$

where I is unit matrix, and after transformation, we have

$$A = C^{-1} \cdot B = C^{-1/2} \cdot C^{-1/2} \cdot B \cdot C^{-1/2} \cdot C^{1/2} = C^{-1/2} \cdot U \cdot C^{1/2} \tag{12}$$

where matrix U has the form

$$U = C^{-1/2} \cdot B \cdot C^{-1/2} \tag{13}$$

and its elements are

$$u_{ij} = (C_i C_j)^{-1/2} b_{ij}; \; i \neq j \tag{14}$$

$$u_{ii} = C_i^{-1} b_{ii} \tag{15}$$

Matrix U is symmetrical matrix and its eigenvalues are real.

Because matrix A is similar to U, the eigenvalues s_i, s_2, …, s_N of matrix A are also real.

Let us construct a quadratic form

$$x^{T} \cdot U \cdot x = -\sum_{i=1}^{N} \frac{b_{i0}}{C_i} x_i^2 - \sum_{i=j+1}^{N} b_{ij} \, (x_i C_i^{-1/2} - x_j C_j^{-1/2})^2 \tag{16}$$

assuming, that

$$x^{T} = [x_1, x_2, \ldots, x_N] \tag{17}$$

With respect to heat exchange between the system and environment

$$\sum_{i=1}^{N} G_{i0} > 0 \tag{18}$$

and at least one of coefficients b_{i0} is not equal zero. Whereas matrix U is negatively defined and all its eigenvalues are negative and various. Because matrix A is similar to it, thus its eigenvalues are negative and various.

According to time constant definition τ_i of system, $\tau_i = -1/s_i$, time constants τ_i are real, positive and various. Thus it results, that the general multi-body heat balance equation described a stable system, which has different time constants.

This corresponds to the conclusion obtained from solution of Fourier equation, having a finited number of time constants.

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Zusammenfassung — Vorliegend wird gezeigt, daß die allgemeine Wärmebilanzgleichung für Mehrkörpersysteme ein stabiles System mit verschiedenen Zeitkonstanten darstellt. Die mathematischen Grundlagen der Mehrkörpermethode werden ebenfalls gegeben.